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INVESTIGATION OF MATERIAL PERCOLATION CAPABILITY WITH USAGE OF DYNAMIC MODELS*

ИССЛЕДОВАНИЕ ПРОТЕКАНИЯ ПОРИСТЫХ МАТЕРИАЛОВ С ПРИМЕНЕНИЕМ ДИНАМИЧЕСКИХ МОДЕЛЕЙ*

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In this article we will use a network terminology for our percolation model description. Although we will be investigating the propagation of the liquid or gas through porous material whereas this material dynamically stretches and shrinks, we will use term “signal” as denotation of percolating substance, and term “network” as denotation of porous material.

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В статье демонстрируется применение перколяционной модели для изучения просачивания жидкости или газа через пористый материал при условии динамического открытия и закрытия пор (например, если материал динамически растягивают или сжимают). Мы показываем, что существующие перколяционные модели требуют существенной коррекции для того, чтобы с их помощью стало возможным моделирование процессов в динамических системах. Предложен вариант сведения ситуации динамической системы к ранее известным моделям.

Keywords: percolation, dynamic models, computer simulation.

Ключевые слова: протекание, динамические модели, имитационное моделирование.

Various kinds of percolation models are widely used now in many applications, for example see [1]. The concepts of percolation theory are the following. The infinite graph is taken as a basis. Often, it is simply regular lattice. The random subgraph of this graph is allocated. According to the “nodes model” each vertex could have “conductible” state. This state is randomly and independently assigned with probability p. Subgraph consists of only conductible vertexes and adjacent is any neighbor vertex of the base graph with conductible state. The main research object is the structure of connected components. The most notable effect is percolation change of phase with parameter p. In many cases the percolation threshold p* exist: if p<p* almost surely all connected components are finite and if p>p* almost surely infinite component exists (percolation cluster). When percolation cluster exists it is said that system percolate.

We study, in some aspect, the interaction ("signal" transfer) in a set of moving objects. For our researches we’re considering a discrete percolation models. Moreover, considering specificity of object the “links model” where condition is set for edges but not for vertexes is more suitable. Of course simulation was made with big enough, but, finite graphs. In this case the percolation threshold is a value of probability p* at which «percolation cluster» - the coherent area that providing signal transmission from one border to another is formed.

The feature of all models mentioned above is the independence of node or link state assigning. Apparently in our extensions of the percolation models this condition is not met. In the first part we show that percolation effect exists in the dynamic networks. In the second part the existence and influence of dependences between the links is proved. In the third part we consider the impact of one of signal transmission parameters (life time) on a transmission in such models and propose the possible approach for reduction of dynamic system models to earlier researched models with independent states.

1. Signal Propagation in the Networks with switching links

1.1 Model Description

To test our assumption that percolation effect exists in the dynamic networks we propose a dynamic links network model. We run the signal transmission simulation and gather the run statistics. The aim is to study the probability of the signal propagation through the system. As a key parameter we consider the concentration of active links – current ratio between number of active links and all links.

A square lattice of NxM nodes is given. L - the number of links. At each moment the connection is determined by Walsh functions. Quick overview of Walsh functions is made in [2] and more detailed in [3].

Model notation: wi – Walsh function. Function w0 is the constant 1. If the function has a value 1 the link is active and inactive otherwise. Thus, if the connection is modeled with function (-w0) the link is inactive during the entire cycle. Time takes discrete values $t_i \in [0,1]$, i=0,...,I.

Modeling tool is to build a synchronized clusters cli, i = 0 ... I. Cluster cli is the subset of synchronized links. For each link the cluster is chosen randomly with conditions:

$$\left|c\ell_i\right| = 2n_i,\ \sum_{i=0}^{I}(2n_i) = L .$$

Current connectivity of the network is determined by the following rules:

1) for each cluster except cl0 ni links operate in a wi regime (direct mode);

2) the remaining links of the cluster work in the inversion mode, ie (-wi).

Thus, for non-zero cluster at each time point ni links (50%) are active. The mode for links is chosen randomly. Links states within the cluster are synchronized. Different clusters are switched independently, since the corresponding Walsh functions are orthogonal.

To control the amount of active links in whole network cl0 cluster size is an input parameter. Also, the input parameter is the relative number of nodes in the cluster cl0 which operate in the direct mode. The size of the remaining clusters is selected so that they have the equal size and cover the entire network. The conditions which we set for our network model provide a constant global network parameter – the percentage of active connections. Thus, the propagation of the signal with the time-to-live is achieved with local rearrangements. The transfer process terminates in two cases: 1) the simulation time exceeds the I; 2) the signal reaches the destination – last layer.

1.2 Simulation and Results

Simulation was performed on the Parallel Network Simulator that was developed earlier and installed on high performance cluster system [4]. We run a simulation on a square lattice with size 300x300. The cl0 cluster size was 40% of all links. Number of clusters I=8. The number of direct links in zero cluster was 20, 25 and 27% of the links in the cl0. To gather statistics, we made 1000 simulation runs.

As a result of the simulation we obtained the probability of signal propagation in the system. On the Fig. 1 (probability of signal propagation on a square lattice with size 300x300) the n is the layer number and pn is the relative frequency of signal receiving. For 27% of direct links the signal percolates through all layers.

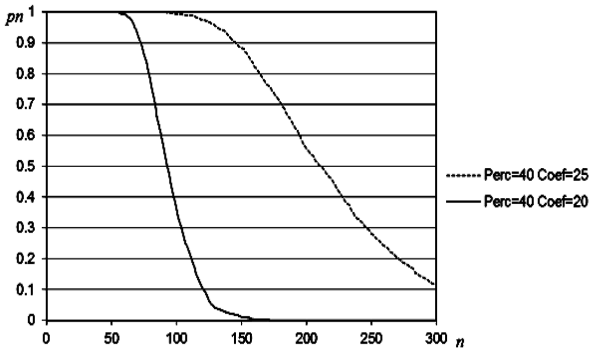


Fig. 1

The results allow us to conclude that probability of signal propagation increased dramatically even for small increase of active links number. Please note that these results may be differ depending on concrete link switching pattern. In this chapter a link switching pattern is very synthetic. In the next chapter we will investigate signal propagation in the networks, in which a link switching is caused by peer movement.

2. Determination of the Percolation Threshold in a System of Moving Objects

2.1 Model Description

Taking into account a role which is played by cyclic periodic and almost periodic processes, we consider a problem of signal transmission in the system consisting of objects, moving cyclically (on a circle). This part is devoted to deriving a threshold value of probability p* of link presence between two neighbor objects at which the probability of signal transmission through the system changes instantly. Certainly, problem statement supposes any generalizations.

Suppose that in nodes of a flat rectangular lattice with step d the centers of circles are located. All circles has the same radius r (d>2r) and objects rotate on these circles with incommensurable speeds. The signal is sent to all objects (1, j), and it is considered the transmitted through system if it will reach at least one of objects $(n, j), 1 \le j \le m$.

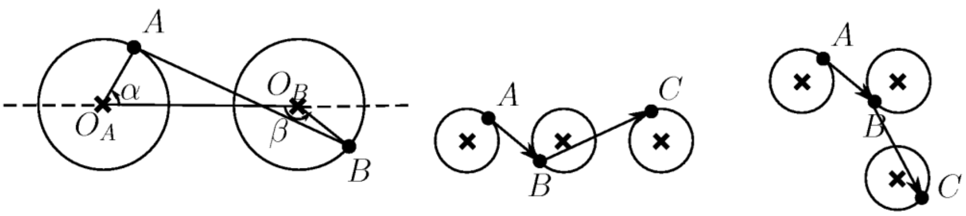
The signal can be transmitted instantly and without a delay from object (i, j) only to the neighbors $(i, j+1), (i-1, j), (i+1, j)$ if the following condition is met: the distance between objects of communication is less than k, $(d-2r<k<d+2r)$. Unlike a classical percolation case (see a review in [1]), signal transmission possibility between two objects depends on time.

Configuration space of system is $m \cdot n$ dimensions torus. System evolution is described by a trajectory on torus $\bar{\phi}(t)=\bar{\phi}(0)+\bar{\omega}t$, where $\bar{\phi}(0)$ – a vector of initial phases of objects, and $\bar{\omega}$ – a vector of frequencies of rotation of objects on circles. As frequencies of rotation are incommensurable, on a consequence from the theorem of averaging [5, p. 248] or more detailed in [6] the following equality is fair:

$$\lim_{T\to\infty}\frac{1}{T}\tau(T)=\frac{1}{(2\pi)^{mn}}|D|, \quad (1)$$

where D – Jordan domain on torus, $|D|$ – its measure, $\tau(T)$ – time during which for an interval [0, T] the trajectory is in D. The probability of a finding of system in condition D is proportional to measure D. Using (1), we will find a probability p from which the signal can be transmitted between two neighbor objects. Let area D on two-dimensional torus is set by a condition (fig. 2-a):

$$|AB|^2=r^2(\sin\alpha-\sin\beta)^2+(d-r\cos\alpha-r\cos\beta)^2\le k^2. \quad (2)$$



a) b) c)

Fig. 2

Thus the probability of link presence can be found as

$$p=\frac{1}{(2\pi)^2}\iint_D d\alpha d\beta. \quad (3)$$

With $d=3, r=1, k=3$ the formula (3) gives p=0.4316.

Events of signal transmission from A to B and from B to C (fig. 2-b and fig. 2-c) are not independent. (Figure 2 – peer configurations: a) parameters of D domain; b), c) peer location configurations 1 and 2). So the direct calculations with $d=3, r=1, k=3$ shows that the probability of a transmission of a signal from A to C on fig. 2-b is equal 0.1098, but the product of probabilities of a signal transmission from A to B and from B to C is $0.4316^2=0.1863$ For a configuration on fig. 2-c results are closer – 0.1866.

2.2 Simulation and Results

Process of signal transmission through system ("run") is modeled. The following assumptions are made. Each object of the system is interpreted as node which moving with certain rule and receiving/sending signals. Suppose that the signal propagates in the system instantly and nodes do not change the position during a run. Also a link between two nodes can be used only once. The purpose of the modeling is definition of critical value of probability p* at which the probability of signal transmission through system changes from zero to one instantly.

As in the first part we use the Parallel Network Simulator for high performance computations. Its modular architecture allows us to make simulations with different network models and propagation rules. For this simulation following extension modules were developed:

1) The initialization module. This module randomly locates each node on a corresponding circle. Input parameters: d - a lattice step, r - radius of circles, N - quantity of nodes per layer.

2) The network topology generation module. For each node this module checks a connection possibility(|AB| < k) with neighbor nodes. If condition is met then nodes are added to the list of available nodes. Input parameter is k max distance of communication.

3) The main module. This module transfers packages according to a model logic and checks conditions of simulation end: a) The Package is delivered to last layer. We consider this run successful; b) It is impossible to transfer any package.

A set of simulation runs are performed to gather statistics. During a simulation the probability of a signal transmission through system is estimated and the relation of number of successful runs to total is calculated. Parameters of simulation are changing to find out, when there is a qualitative reorganization of a signal transmission process. The simulation was performed for a square lattices with sizes $N \times N$ (N=100,300,500). Input parameters (see above), used in experiment: d, r are constant (d=3, r=1); k changes in some range. Under an axis k, values p are specified (probability of a signal transmission between pair of nodes). They are defined by parameters of simulation and calculated according to the formula (3). Those probabilities which correspond to some characteristic values k are specified only. Results are reflected in the fig. 3 (the dependence of the probability estimates for the signal transmission through system).

With sufficient accuracy the existence of the percolation threshold is estimated. The signal transmits through the system with p = 0.6. The resulting value differs on 20% from the value 0.5, which is the percolation threshold for the case of percolation on a square lattice described in [7]. The reason for this effect is the dependencies between links. That leads to a significant change in threshold value.

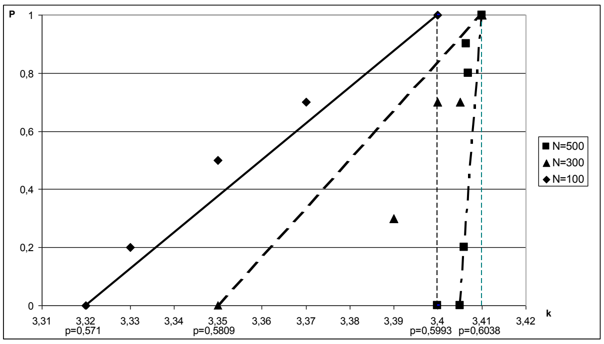


Fig. 3

From this we can conclude that the existing percolation models, at least, applied without a significant correction would significantly distort the picture of the signal transmission in the dynamic systems which model moving objects.

Conclusions

We propose the dynamic percolation system model with link states determined by Walsh functions. The simulation allows us to conclude the existence of percolation effect in the dynamic networks. We perform the simulation to determine the percolation threshold for model with cyclic movements dynamic. Results of numerical experiment with sufficient degree of accuracy have allowed to establish percolation threshold existence (it is equal approximately 0,6) and differs (on 20 %) from corresponding value for static system.

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