UDK 677.022:519.24

COMPUTER MODELING OF STRAND ELONGATION

P.A. SEVOSTYANOV

(The Moscow State Textile University named after A.N. Kosygin)

Computer models are considered, which allow to forecast dependence between the elongation of a strand of homogeneous threads and the strength of their strain (so called strain – strength (S-S) curves), if this dependence is known for individual threads of a strand and with consideration for statistical dispersion of breaking load and breaking elongation load of threads.

In the case, when deformation of threads is described by Hooke's law, and all threads have an identical elasticity module k, dependence of strain strength of a strand P(S) from its elongation S can be found analytically [1]. With elongation of the strand S, load on each not ruptured fiber will equal to kS. Number of ruptured fibers in a strand equals to

$$n_b = N\int_0^{kS} f(p_{max})dp_{max} = NF(kS), \quad (1)$$

where $f(p_{max})$, $F(p_{max})$ – density and function of breaking load distribution p_{max} of threads, N – initial number of threads in a strand, and dependence of strain strength of a strand from its elongation

$$P(S) = \left(N - n_b(S)\right)kS = NkS\left(1 - F(kS)\right). \quad (2)$$

Elongation value S_{max}, at which strength of strand resistance to elongation is maximal, can be obtained from the equation

$$dP(S)/dS\big|_{S=S_{max}} = Nk\left(1 - F(kS_{max})\right) - Nk^2 f(kS_{max}) = 0. \quad (3)$$

By substituting the obtained value S_{max} in (2), we will get maximum resistance strength $P_{max} = P_s(S_{max})$, generated by a strand during its stretching. The determined value P_{max} allows to evaluate yarn strength factor $\eta_{max}=P_{max}/(Np_{sr})$ by their average strength p_{sr}.

Below is a list of directions of model complexification: 1) difference of elastic deformation from Hooke's law; 2) inhomogeneity of yarn in a strand; 3) correlation between breaking elongation and yarn load; 4) statistical dispersion of parameters and characteristics of yarn deformation and its manifestation in strand's properties; 5) modeling of different conditions of strand stretching; 6) considering inelastic components of deformation and dynamic, for instance, pulsating loads.

Any complexification of a model should be justified from the point of view of its manifestation in the results with consideration for statistical dispersion of parameters. In order to ensure continuity between models and possibilities of endless complexification, it is advisable to switch over from analytical models to computer modeling of behavior of a strand during stretching [2]. For this purpose, modeling algorithm has been developed implemented in software, in which first four directions of the above mentioned list have been implemented.

In most of natural and chemical yarns, dependence p = φ(S) differs from a linear one, corresponding to Hooke's law [1]. In case of computer modeling, it is more suitable, as a rule, to approximate this dependence (obtained in a real experiment) by spline functions. In Fig. 2 this dependence is given for orlon fibers [3] and its approximation by polynomials of the 3rd, 4th and 5th order and cubic spline. Approximation by spline function appears to be more exact and is naturally incorporated into the modeling program. Let us indicate dependence being typical for a concrete type of fibers as $p_c = \varphi_c(S_c)$. Then with consideration for a real range of change of p and S from zero to p_{max} and S_{max} dependence for a concrete thread can be represented as follows:

$$p = \varphi(s) = p_{max}\varphi_c\left(S/S_{max}\right). \qquad (4)$$

In the simplest case of modeling, values p_{max} and S_{max} for a concrete thread are considered as independent and are random values with certain distribution laws. There is interest to define the effect of these random variations S_{max} and p_{max}, as well as deviations of dependence (4) from the linear Hooke's law on S-S – curves of yarn strand. To compare results of modeling, it is more convenient, instead of absolute values of strain P of a strand, to use relative specific tension of the thread η(S) and a portion of ruptured threads from their initial quantity a_b:

$$\eta(S) = P(S)/Np_{sr}\,;\ a_b = n_b/N. \qquad (5)$$

In model p_{max} ~ Norm(m = 5; CV = 20%); S_{max} ~ Norm(m = 4%; CV =25%); N = 200.

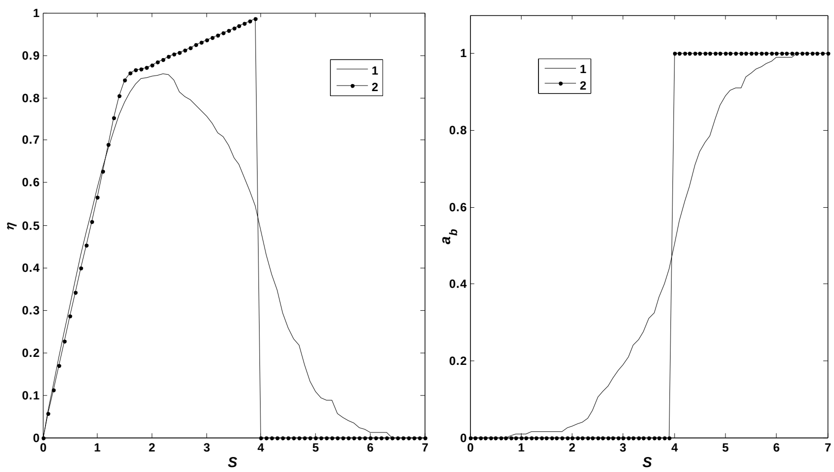


Fig. 1

The curves in Fig.1 demonstrate results of operation of the model. The curves 2 show results with CV S_{max} = 0. In this case dependence η (S) represents exactly the S-S – curve of yarn (Fig. 2). Threads rupture simultaneously, reaching breaking elongation mS_{max}=4%.

Scattering of values of yarn breaking strain with the coefficient of variation CVp_{max}=20% has practically no impact on the form of the curve, which is manifestation of the law of averages for large numbers (N = 200) of threads per strand. In fact, if N will be reduced to 10, then each new simulating of model will generate a curve of dependence η(S), which differs from the remaining ones.

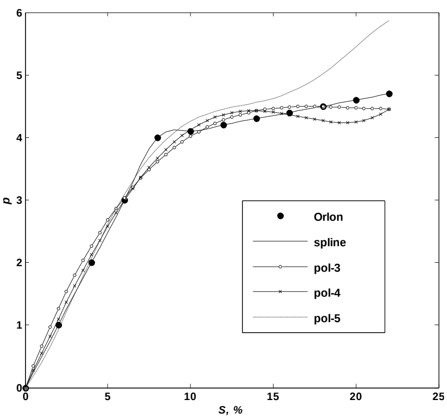


Fig. 2

Nevertheless, all these curves overlap with the accuracy up to the scale multiplier. In Fig. 3

curve 2 for N = 200 and curves 3…7 on five independent runs of simulating of a model with N = 10 are shown, which confirm the conclusion made.

On the contrary, scattering of values of breaking elongation of threads influences significantly the character of dependence η (S). Fig. 4 shows dependences η (S) and $a_b(S)$ with different values of variation coefficient CVS_{max}: 0%, 5%, 10% and 25%. Increase of scattering in values of breking elongation of threads of a strand extends the curve η(S) along the X-axis, simultaneously decreasing coefficient of utilization of yarn strength η_{max}.

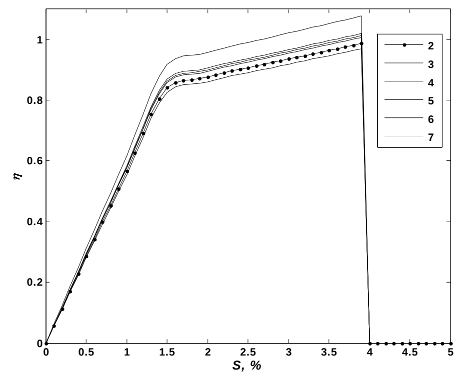


Fig. 3

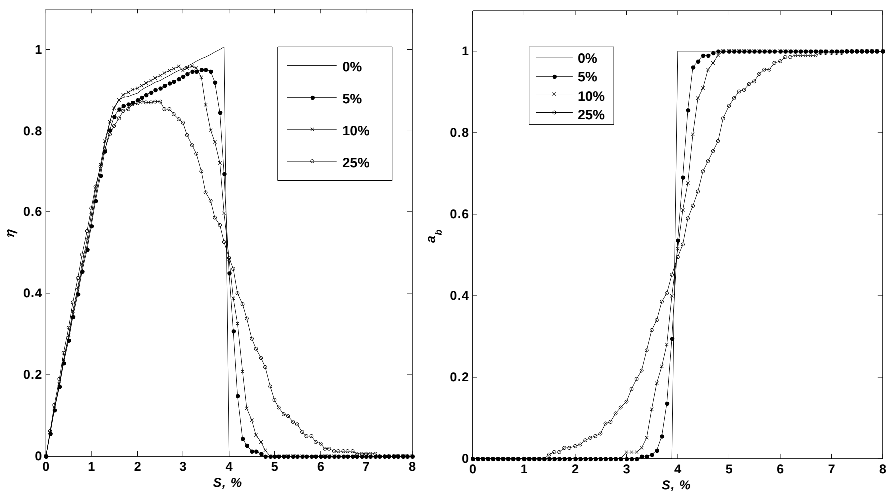
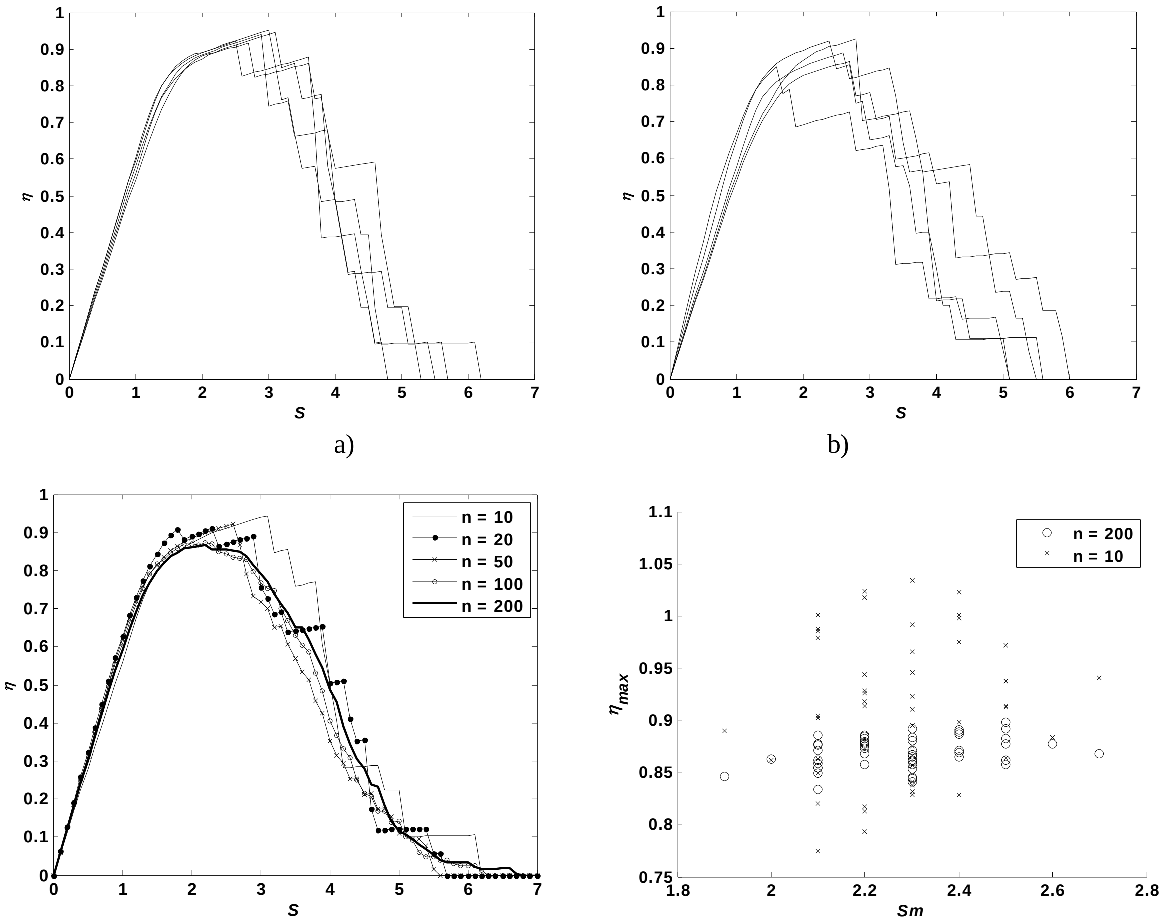


Fig. 4

Rupture of threads happens not simultaneously, but with different values of stretching of a strand, thus transforming from a single event into a process of gradual destruction, extended in time. Similar situation is observed for all homogeneous types of yarn, independently from specific of their S-S – curves.

With a small number of threads in a strand, for instance, N = 10, random scattering of values p_{max} and S_{max} will take place as a result of modeling. In Fig. 5-a assemblage of dependences, obtained with CVp_{max} = 0% and CVS_{max} = 25% with different realizations of random values p_{max} and S_{max} is shown. Results in Fig. 5-b differ only in the fact that they are obtained with CVp_{max} = 20%. Increase of scattering of yarn breaking load resulted in big difference in dependences with preservation of their general view.

By comparing dependences in Fig. 4 and 5, one can see that decrease of number of threads in a strand leads to appearance of characteristic step-like signals, which are related to break of concrete threads. With big number of threads in the stand, these areas are smoothed, since the number of "steps" increases, and their relative height decreases, and they become practically invisible on the graph. In order to confirm the aforesaid, in Fig.5-c graphs of dependence η (S) are given, obtained with fixed values CVp_{max} = 20% and CVS_{max} = 25% and different values n = 10; 20; 50; 100; 200. The curves are obtained in different realizations of values of random values p_{max} and S_{max}.



a) b) c) d)

Fig.5

Probability scattering of values p_{max} and S_{max} leads to scattering of values of utilization coefficient of yarn strength in the strand η_{max} and the appropriate elongation of the strand, which we will denote with S_m. These values are determined by formulas

$$\eta_{max} = \max_{S>0} \eta(S), \qquad S_m = \arg\max_{S>0} \eta(S). \tag{6}$$

Fig. 5-d shows scattering graphs (S_m; η_{max}) obtained from 50 repeated simulating models with N = 10 and N = 200. With increase of quantity of threads in the strand, scattering η_{max} decreases significantly. Practically, there is no dependence of scattering of elongation values, when the maximum value η_{max} is observed, from the number of threads per strand. Portion of sample values η_{max}, being accounted for a certain value S_m, remains practically the same, irrespective of N.

Real yarn has correlation between breaking elongation S_{max} and breaking strain P_{max}, and one-dimensional distributions of these characteristics differ from a normal distribution.

When modeling is carried out, considering peculiarities of these yarns presents no problems. It is found that presence or absence of correlation of both signs between the breaking elongation and breaking strain will lead to differences only within the statistical dispersion of results, i.e. it does not influence the S-S – curves of the strand. Yarn deformation law along with the distribution law S_{max} and p_{max} significantly influences S-S – curve of yarn strand deformation. Difference not only in extremum values P_{max} and S_{max} are observed, but also in the shape of a curve. This way, during modeling of elongation of the strand of threads it is extremely important to correctly set up the law of deformation of individual threads both by a shape of a deformation curve, as well as by values of statistical dispersion of values.

B I B L I O G R A P H Y

1. *Morton W.E.*, *Hearle J.W.S.* Physical properties of textile fibers. – Manchester – London, 1962, (transl. from Eng.), Legkaya Industriya, 1971.
2. *Sevostyanov P.A.* Computer modeling of technological systems and spinning products. – M.: Inform-Znaniye, 2006.
3. *Gusev V.E.* Chemical fibers in textile industry. Legkaya Industriya, 1971.

Recommended by the department of information technologies and systems of automatic design. Received 05.06.09.