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PREDICTION OF THERMAL CONDUCTIVITY OF A COMPOSITE TEXTILE MATERIAL

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In conformity with standards НПБ 157-99 [1] the fire-fighter's clothing (FFC) possessing dehydrating, thermo-insulating and fire-proof properties is manufactured from laminated composite materials supplied with detachable thermo-insulating linings. The linings are fabricated using 2 layers of a woollen needle-punched non-woven textile fabric and because of their bulky dimensions have limited applications. In order to increase the thermo-insulating properties of the basic FFC outfit it is suggested to introduce an internal stratum of a knitted material. In this case the best choice is the warp-knitted material made of flax yarns, as it is characterized by an equal extensibility in all directions and add some ergonomic properties to this outfit. In their turn the flax fibres possess high thermo-insulating and heat-resistant properties enhanced by the cellular porous structure of the knitted fabric. In the critical zones, i.e. in the places of contact with the heated objects (as a rule they are shoulder girdle, chest, thighs, knees), the knitted stratum can be fortified by tightening the structure or introducing the weft yarns. Apart from this the yarns produced from flax wastes are rather inexpensive.

In order to design such an outfit we must calculate thermal conductivity of the sandwich shell taking into consideration thermo-insulating properties of the fibres, structure of the knitted material, heat released by a man, external thermal field. Such problem can be solved using the equations of thermal conductivity employing the method of finite elements.

The shell being designed as well as a human figure can be regarded as axis-symmetric and the problem can be solved in a polar coordinate system (Fig. 1).

There is a well-known Laplace's equation [2] for determining the thermal conductivity

$$\Delta^2 T = \frac{\partial^2 T}{\partial r} + \frac{\partial^2 T}{\partial z} = 0\,, \tag{1}$$

where Δ is the Laplacian; T – the temperature; r and z are the polar coordinates.

The problem is given for a variety of elements l consisting of the regions with boundaries S; Dirihle boundary conditions on a boundary segment will be:

$$T = 36.6°\ C,\ z = 0,$$
$$T = 200°\ C,\ z = H$$

and the Neuman conditions for the rest of the boundary will be:

$$\frac{\partial T}{\partial r} = 0,\ r = 0,$$
$$\frac{\partial T}{\partial r} = 0,\ r = L$$

36.6° C here is the temperature of a human body within the shell, 200°C – the most likely temperature of the radiant heat flux outside the shell.

The problem is solved with the help of the equivalent variation formulation whereby the solution T (r, z) coincide with certain function I (r, z) which is minimized with the help of the functional

$$I = \frac{1}{2}\iint_S \left[\left(\frac{\partial \widehat{T}}{\partial r}\right)^2 + \left(\frac{\partial \widehat{T}}{\partial z}\right)^2\right] drdz\,, \tag{2}$$

where $\widehat{T}(r,z)$ is the function defined from the admissible variety of sampling functions specified for the surface *S*. The continuous functions having first piecewise continuous derivatives and satisfying the above said boundary conditions are considered as admissible. These conditions will comprise a set of control functions and dependences known from heat engineering:

$$\begin{aligned} I = \iint_S \frac{1}{2}\Bigg[& \lambda_r\left(S_\mu, T\right)\left(\frac{\partial T}{\partial r}\right)^2 + \lambda_z\left(S_\mu, T\right)\left(\frac{\partial T}{\partial z}\right)^2 + \lambda_r\lambda_z\left(S_\mu, T\right)\frac{\partial T}{\partial r}\frac{\partial T}{\partial z} - \\ & -2Q\left(S_\mu\right)\Phi_Q(t)T + 2\rho_c\left(S_\mu, T\right)\frac{\partial T}{\partial t}T\Bigg] rdS - \int_L q(L,)\Phi_q(t)TrdL + \\ & + \frac{1}{2}\int_L \alpha(L)\Phi_\alpha(t)\left[T - 2T_b(L)\Phi_T(t)\right]TrdL + \frac{1}{2}\int_L \alpha(L_k,\sigma_k)\Phi_\alpha(t)\left[T - 2T_b(L)\Phi_T(t)\right]TrdL) + \\ & + \int_L \varepsilon(L)\Phi_\varepsilon(t)\left\{\frac{T^5}{5} - \left[T_u(L)\Phi_u(t)\right]^4 T\right\} rdL, \end{aligned} \tag{3}$$

where λ_r, λ_z, λ_{rz} are the heat conductivity coefficients:

$$\lambda = \frac{q\delta}{\left(T_1 - T_2\right)St}$$

q is the intensity of the heat flow passing through the boundary; t – the time factor, δ – the thickness of the material; Q – the intensity of the internal sources of heat (heat released by a human body); ρ_c – the specific heat capacity of the material per a volume unit; αT_b – the heat transfer coefficient and the ambient temperature in the boundary region; ε, T_u – the radiant heat-transfer coefficient and the radiant temperature; $\Phi_Q(t)$, $\Phi_q(t)$, $\Phi_\alpha(t)$, $\Phi_T(t)$, $\Phi_\varepsilon(t)$, $\Phi_u(t)$ – the control functions**,** correlating with time variable and specified for boundary segments for changing the boundary conditions; S_μ - sub-regions of different materials of region S under consideration; σ_k - contact voltages in the region of contact L_k.

The thermal field is represented as a matrix indicating the temperature in the nodal locations around the shell under study in polar coordinates (Fig. 1). The control functions are also defined as matrices.

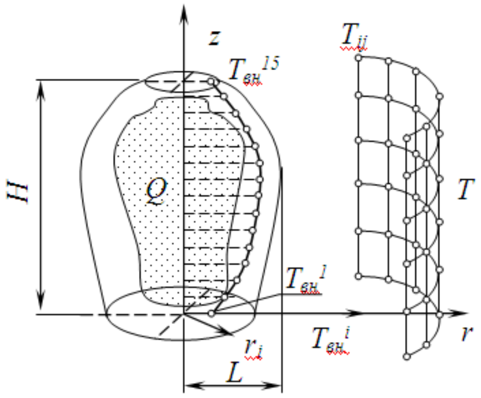

Fig 1. The calculated shell in the thermal field

The whole region is subdivided into finite elements l in a six-chain form, in their shape and size similar to the element of a looped structure of the warp knitted fabric [3]. The subdivision of the region and continuity conditions make it possible to write the functional (3) as:

$$I = \sum_{i=1}^{l} I^e , \qquad (4)$$

where I^e is the functional (3) for the finite element i with the number of nodes e.

The typical e-element is shown in Fig. 2.

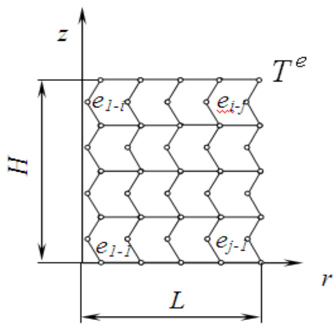

Fig 2. Subdivision of the region into finite element ℓ

The temperature within the shell under stationary heating conditions is determined by the ratio:

$$\frac{\partial I^e}{\partial T_{int}{}^e} = k^e T_{int}{}^e = 0 , \qquad (5)$$

where $T_{int}{}^e$ is the value of temperature in the mesh nodes (Fig. 2); k^e – the stiffness matrix of the element e comprised of the thermal conductivity coefficients [3], [4].

In line with the rules of the finite elements method, for the shell under study a global matrix can be assembled from rows i and columns j in compliance with the expressions (3), (4) and (5):

$$\begin{vmatrix} \dfrac{\partial I^1}{\partial T^{1-1}} \\ \dots \\ \dfrac{\partial I^l}{\partial T^{i-j}} \end{vmatrix} = \frac{1}{l\Delta} \begin{vmatrix} k_1^1 & \dots & k_i^1 \\ \dots & \dots & \dots \\ k_j^1 & \dots & k_j^l \end{vmatrix} \cdot \begin{vmatrix} T_{BH}{}^1 \\ \dots \\ T_{BH}{}^l \end{vmatrix} . \qquad (6)$$

To calculate the thermal conductivity and design the distribution of the thermal fields within the shell a matrix mathematical system MatLab is employed. In programming of the design procedure the rows of matrix equation (6) corresponding to each node were successively calculated in accordance with expression (4).

Let us consider as our example a sample of the material with a number of nodes 15×15. The coefficient of thermal conductivity for flax yarn is 0.04, for cotton yarn – 0.05 for the protective layer of FFC – 0.3 W/(m·°C). Let us assume that the thermal field is homogeneous T=200°C and the control functions are constant, Q – 700 W/m², q – 350 MJ/m², ρ_c – specific heat capacity of the material – $9.18 \cdot 10^5$ J/m³·°C. Then the matrix equation of the system can be written as:

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 0 & 0 & 10 & -2 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -8 & 0 & -2 & 20 & -2 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & -2 & 10 & 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 & 10 & -2 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8 & 0 & -2 & 20 & -2 & 0 & -8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 & -2 & 10 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 10 & -2 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8 & 0 & -2 & 20 & -2 & 0 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & -2 & 10 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \\ T_{13} \\ T_{14} \\ T_{15} \end{vmatrix} = \begin{vmatrix} 36,6 \\ 37,0 \\ 37,5 \\ 38,0 \\ 39,0 \\ 39,5 \\ 40,0 \\ 41,0 \\ 41,5 \\ 40,5 \\ 39,5 \\ 39,0 \\ 38,0 \\ 37,0 \\ 36,6 \end{vmatrix} \tag{7}$$

Temperature within the shell curve is shown in Fig. 1.

C O N C L U S I O N S

Methods are suggested to evaluate the thermal conductivity of the sandwich textile shell subject to the surface area of the upper layer, thermo-insulating properties of each layer, external temperature field, contact and no-contact (irradiation) method of heat transfer, the internal heat source.

The methods suggested can be implemented in designing the heat-proof properties of the fire-fighter's clothing supplied with lining made from warp knitted flax fabric.

B I B L I O G R A P H Y

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