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FORM AND YARN TENSION ON RING SPINNING MACHINES

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The issue regarding path-turning movement of yarn has always been and still is of profound interest for theory and application in the textile technology [1], [2]. Path or steady movement is a term for motion of yarn which maintains continuously its form of a certain unchanged or permanent line. Yarn moves along the line with a preset relative speed $v_r = v_r(t)$, and the line itself is motionless or moves with random order. This motion is implemented on the ring spinning machine with twisting and bobbin winding. Yarn with linear density μ moves with the constant speed v from the drawing mechanism through the thread guide A (fig. 1), then it passes through the traveller B which moves with a constant angular velocity ω along the ring K of radius $R_к$, and then it is wound on the bobbin of radius r.

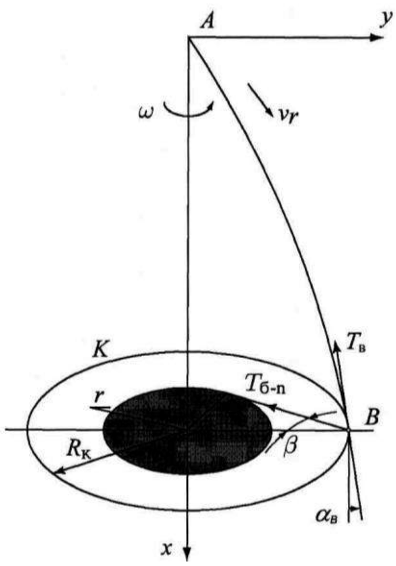

Fig. 1. Baloon on a ring spinning machine

In traditional ring spinning, rotational speed reaches 20000 min^{-1}. With such high angular velocities, yarn forms a visible surface, which is called balloon. Winding on ring spinning machines with participation of yarn, which rotates around its axle and simultaneously moves along a certain unchanged line, can be brought to path movement of yarn in its individual case – to the condition which "looks like standstill". Correct solutions to the problem of ballooning of yarn are given in [1]…[5], [8]. A relatively complete bibliography can be found in [3]. However, the papers of renowned or even prominent mechanic-engineers listed in the mentioned bibliography are restricted by investigation of movement or relative equilibrium of yarn at the yarn end of travelers, i.e. weight mass $m_в$ moving on the ring with friction. The mathematical model of balloon constructed in [6] considers quite complete conditions, but accepted assumptions, inaccuracies, as well as outdated modeling method restrict accurate information regarding winding by means of a ring spinning machine.

With uniform movement of yarn along the seems-to-be-standstill line, when outside forces are present, the seems-to-be-standstill line overlaps with the line of equilibrium of yarn with the same forces and with the same boundary conditions and linear dimensions. Tension of moving yarn increases in this case by value μv^2 [3]. Let us quote a numerical estimate of μv^2. With linear density 25 tex, rotation velocity of spindle 20,000 min^{-1}, 810 twists per meter and, consequently, path velocity 24.7 m/min μv^2 value is only $4.2 \cdot 10^{-3}$ mN. Then nonstretchable yarn with linear density μ, turning together with coordinate system Oxy with angular speed ω around the fixed axle x can be considered as being at rest without changing actual tension T for apparent tension $T^*=T-\mu v^2$.

Computation using mathematical balloon model, considering quite general conditions, indicates that the task can be significantly simplified. Usually the following is neglected: first, gravity, second, air resistance. During further simplification it is assumed to disregard Coriolis force (it is small in comparison to force of moving space because of $v_r << r\omega$). Calculations show that these assumptions do not add poor accuracy to computations.

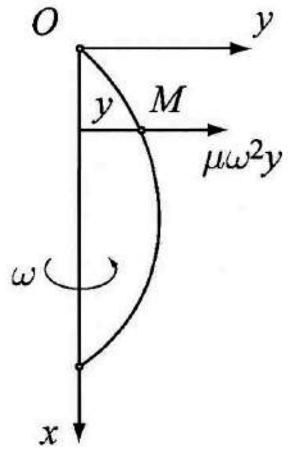

Fig. 2. Relative equilibrium of the ratating thread

One of variants reduces the mathematical model of balloon to the problem of turning yarn, which is attached with ends on the turning axle (Fig. 2). P. Appel, who considered the problem of finding the equilibrium position of weightless yarn, turning with a constant angular velocity around the axle (problem on "jumper") demonstrated that yarn form in this case is expressed in elliptic functions [4].

Let us project basic equation of path movement on axes of coordinates which are turning with yarn Oxy [3], [5]. We will get two equations of path movement of turning yarn

$$\frac{d}{ds}\left(T\frac{dx}{ds}\right)=0,\quad \frac{d}{ds}\left(T\frac{dy}{ds}\right)=-\mu\omega^2 y. \quad (1)$$

From the first equation of the system (1) it follows that projection of yarn tension on the direction of rotation axis is uniform along the whole length of yarn:

$$T\frac{dx}{ds}=C=\text{const}.$$

From here we will obtain tension T and place this value into the second equation:

$$\frac{d}{ds}\left(Cy'\right)=-\mu\omega^2 y.$$

Here $y'=\frac{dy}{dx}$. We enter designation

$$\frac{\mu\omega^2}{C}=\frac{2}{a^2},$$

considering that

$$ds=\sqrt{1+y'^2}\,dx=\frac{\sqrt{1+y'^2}}{y'}dy.$$

Then the previous equation will be written as

$$\frac{y'dy'}{\sqrt{1+y'^2}}=-\frac{2y}{a^2}dy.$$

After integration we will get

$$\sqrt{1+y'^2}=\frac{b^2}{a^2}-\frac{y^2}{a^2},$$

where b – new constant of integration. If we solve the equation for y', we will obtain

$$\frac{dy}{dx}=\pm\frac{1}{a^2}\sqrt{\left(b^2-a^2\right)^2-a^4}.$$

By dividing variables and integrating, we will get

$$x=\pm a^2\int\frac{dy}{\sqrt{\left(b^2-a^2-y^2\right)\left(b^2+a^2-y^2\right)}}. \quad (2)$$

Itegral is expressed by special functions which are called elliptic functions. Then all critical points are located in the class of elliptic sine [4]:

$$y=\sqrt{b^2-a^2}\,\mathrm{sn}\left(x\frac{\sqrt{2}}{ak'}\right),\quad k'=1-k^2,\quad k^2=\frac{b^2-a^2}{b^2+a^2}. \tag{3}$$

Value a with data l and h can take calculational set of values

$$a(n)=\frac{h\sqrt{2}}{2nKk'},\quad n=1,2,\ldots\quad K=\int_0^1\frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}},$$

i.e. there exist a countless number of forms (which consist from one, two, three etc. half-waves) of relative equilibrium of yarn. Stability theory methods on the basis of variational Lagrange equation indicate that there exist the only curve of length l, passing via points O, A which has no more than one half-wave at the section [O, h]. As a result, practically only one stable form of relative equilibrium is realized.

Let us define the unknown parameters a and b. Using equality (2), we will get the first equation

$$\frac{h}{2}=a^2\int_0^{\sqrt{b^2-a^2}}\frac{dy}{\sqrt{\varphi(y,a,b)}}, \tag{4}$$

where $\varphi(y,a,b)$ – polynomial, which stands under the radical sign in (2). Let us obtain linear element

$$ds=\sqrt{1+y'^2}dx=\frac{b^2-a^2}{\sqrt{\varphi(y,a,b)}}dy.$$

Now let us write the second equation

$$\frac{\ell}{2}=\int_0^{\sqrt{b^2-a^2}}\frac{b^2-a^2}{\sqrt{\varphi(y,a,b)}}dy. \tag{5}$$

Then from $\frac{\mu\omega^2}{C}=\frac{2}{a^2}$ we can obtain projection of yarn tension on the direction of rotation axis C. Taking into consideration

$$\frac{dx}{ds}=\frac{1}{\sqrt{1+y'^2}}=\frac{a^2}{b^2-y^2},$$

tension of turning yarn is determined by the formula

$$T=\frac{C}{a^2}\left(b^2-y^2\right). \tag{6}$$

With preset h and ℓ equations (4) and (5) are solved with a computer and unknown parameters a and b are obtained.

Solution can be obtained in analytical form, if yarn is considered quite sloping [1], [3], [8]. Let us assume that length of yarn l differs slightly from the distance h between the points of yarn's fixing. Then the angle α between the tangent to yarn and the rotation axis x is small and derivative $y'=\mathrm{tg}\alpha<<1$.

Let us use approximate relationship, which is obtained on the breakdown of radical with series expansion in powers of

$$y': ds=\sqrt{1+y'^2}dx\approx\left(1+\frac{1}{2}y'^2\right)dx.$$

General solution to the problem for the sloping turning yarn

$$y=\frac{1}{\omega}\sqrt{\frac{C_1C_2}{\mu}}\sin\left(\sqrt{\frac{\mu}{C_1}}\omega x\right) \tag{7}$$

and exact (3) solutions do not differ from each other in quality.

Tension T is determined by the formula

$$T = \frac{\mu\omega^2 h^2}{\pi^2}\left[1 + 2\frac{\ell - h}{h}\cos^2\left(\frac{\pi}{h}x\right)\right]. \quad (8)$$

For balloon between the thread guide and the traveller on the ring spinning machine (Fig. 1) differential equation system and general solution for the sloping turning yarn are valid, in form of (7):

$$y = \frac{1}{\omega}\sqrt{\frac{C_1 C_2}{\mu}}\sin\left(\sqrt{\frac{\mu}{C_1}}\omega x\right).$$

In all turning axes Axy thread carrier A and traveller B have constant coordinates: x_A=0, y_A=0, x_B=h, y_B=$R_к$. But in addition to boundary conditions, additional condition is to be added which will be fundamental; this condition determines significant difference of the balloon problem on the ring spinning machine from the ballooning without mass traveler m_B.

To obtain this condition, let us mentally consider the movement of one ring traveler without yarn, replacing it by a reaction. The following forces will effect the traveler:

– Tension of yarn at the entering point into traveler which is determined by the equation (9) with x=h:

$$T_B = C_1\left\{1 + \frac{1}{2}\left[\sqrt{C_2}\cos\left(\sqrt{\frac{\mu}{C_1}}\omega h\right)\right]^2\right\}. \quad (9)$$

Direction T_B is determined by the value of the derivative at the point B:

$$y' = tg\alpha_B = \sqrt{C_2}\cos\left(\sqrt{\frac{\mu}{C_1}}\omega h\right). \quad (10)$$

– Tension of yarn $T_{б\text{-}п}$ between the traveler and the bobbin. Between T_B and $T_{б\text{-}п}$ a certain functional connection exist which usually is presented in form of Euler formula. Conditions of utilization of this formula result from construction of this widely known formula which is mentioned almost in all courses of theoretical mechanics, and without which textbooks in mechanics of yarn and other fields adjacent to it, cannot go. First of all, it is assumed that yarn is located along the geodesic line, when directions of principal normal of yarn and normal to surface, on which yarn is located, are coincident. If contact of yarn with the traveler would have been along the screw line, finding of $T_{б\text{-}п}$ would not pose difficulties. However, form of the traveler on the contact line is, first, not cylindrical, second, section of traveler's bow is rectangular. Therefore, even if we employ relation between the tight and slack strands of yarn in form of Euler formula, there is no sense to complicate it, and we can restrict ourselves to the simple "flat" case. If branches (strands) of yarn form between each other an angle $\gamma = \alpha_B - \frac{\pi}{2}$, so arc of yarn contact with traveler will be $\pi - \gamma = \frac{\pi}{2} + \alpha_B$. Then we will write as follows:

$$T_B = T_{б\text{-}п} e^{-k_н\left(\frac{\pi}{2} + \alpha_B\right)}. \quad (11)$$

Take note of angle sign α_B. It was noticed earlier that the angle α represents a tangent tilt at the current point of balloon's curve to axle x. Using the right system of reference and calculating angle reference as positive counterclockwise, on Fig. 1 we see that at the origin of coordinates A the angle α_0 is maximal. If the curve of yarn form has extremum at the section between the thread guide and the traveler (between the points A and B), at the point corresponding to y_{max}, $\alpha = 0$, then inclination angle decreases, changing sign to negative one.

– The remaining forces influencing the traveler, as well as conditions of its relative equilibrium are mentioned in courses of manual [5], [6].

From [6] we will write an expression for tension of yarn between the thread guide and the cop:

$$T_{б\text{-}п} = \frac{m_B \omega^2 R_к}{\frac{\sin\beta}{k_к} + \cos\beta + \left(\sin\alpha_B - \cos\alpha_B\right) e^{-k_н\left(\frac{\pi}{2}+\alpha_B\right)}}. \qquad (12)$$

Here $\beta = \arcsin\frac{r}{R_к}$, $k_к$ – coefficient of friction of traveler on the ring.

This way, we can write a system of three independent equations which are solved for C_1, C_2, α_B:

$$R_к = \frac{1}{\omega}\sqrt{\frac{C_1 C_2}{\mu}} \sin\left(\sqrt{\frac{\mu}{C_1}}\omega h\right), \ \mathrm{tg}\alpha_B = \sqrt{C_2}\cos\left(\sqrt{\frac{\mu}{C_1}}\omega h\right),$$

$$\frac{m_B \omega^2 R_к}{\frac{\sin\beta}{k_к} + \cos\beta + \left(\sin\alpha_B - \cos\alpha_B\right) e^{-k_н\left(\frac{\pi}{2}+\alpha_B\right)}} e^{-k_н\left(\frac{\pi}{2}+\alpha_B\right)} = C_1\left\{1 + \frac{1}{2}\left[\sqrt{C_2}\cos\left(\sqrt{\frac{\mu}{C_1}}\omega h\right)\right]^2\right\}.$$

Let us calculate tension $T_{б\text{-}п}$ on the ring spinning machine P-76-5M4 under the following conditions: yarn of linear density μ=25 tex, balloon height h=240 mm, traveler's mass m_B=0.075 g, ring diameter $D_к=2R_к$=45 mm, chuck diameter d=2r=22 mm, rotational speed of spindle 11,000 min^{-1}. The mentioned values of balloon's height h, cop diameter r correspond to the maximum degree of tension between the traveler and the cop under concrete conditions of yarn formation. If we assume $k_к$=0.17, $k_н$=0.23, then we will get C_1=526.6, C_2=0.036, $\alpha_B = -0.062$. Maximum tension during winding on the spinning machine, i.e. tension between the traveler and the cop under the above mentioned conditions will be 74.65 cH. Minimum tension occurs with maximal radius of cop r=20 mm and minimal height of balloon h=112 mm. In this case parameters of balloon's form change notably: angle of yarn entering into traveler is $\alpha_B = 0.145$ radian, projection of yarn tension on the axle x equals to $T\frac{dx}{ds} = C_1 = 324.3$. Then minimum degree of tension between the traveler and the cop is 44.0 cH.

B I B L I O G R A P H Y

1. *Minakov A.P.* On balloon form and yarn tension on silk spinning machines of American and Italian systems. Proceedings of MSTU. – 1927, V.1, issue. 1, p. 1…4.

2. *Minakov A.P.* On balloon form and yarn tension on twisting machines. Proceedings of MSTU. – 1929, V. 2, p. 1…36.

3. Principles of yarn mechanics/Yakubovsky Y.V., Zhivov V.S., Korytysskiy Y.I., Migushov I.I. – M.: Legkaya Industriya, 1973.

4. *Appel P.* Theoretical mechanics. V 1. – M.: Fizmatgiz, 1960.

5. *Merkin D.R.* Introduction to mechanics of flexible yarn. – M.: Science. Main publishing firm of literature in physics and mathematics, 1980.

6. *Sevostyanov A.G., Sevostyanov P.A.* Modeling of technological processes. – M.: Light and Food Industry, 1984.

7. Mechanical technology of textile materials/Sevostyanov A.G., Osmin N.A., Scherbakov V.P. and others. – M.: – Legprombytizdat, 1989.

8. *Scherbakov V.P.* Applied mechanics of yarn. – M.: RIO MSTI named after A.N. Kosygin, 2001.

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