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DETERMINATION OF DYNAMICS MEASURE OF COMPOSITION

E.V. KARSHAKOV, L.B. KARSHAKOVA, A.V. FIRSOV

**(V.A. Trapeznikov Institute of Control Sciences of RAS,
Moscow State Textile University "A.N. Kosygin")**
E-mail: office@msta.ac.ru

Abstract: For automatic sorting of digital images of textile compositions, they must be analyzed in terms of the characteristics of the ornament. In this paper we propose an approach which is based on elementary concepts of classical mechanics and analytic geometry. Definition is based on the mathematical description of the system. Measure of symmetry, as a relation of the number of points without a pair to the total number of points of motif pattern has been introduced. In addition, the parameter of the equilibrium dynamics has been introduced. When considering the concept, problems of determining the characteristics of motif and composition in general were separated.

Digital images of textile compositions, for the purpose of automatic sorting, must be analyzed in terms of the characteristics of the ornament [1]. In this paper we propose an approach based on elementary concepts of classical mechanics and analytic geometry [2], [3]. Modern computers have enough performance to calculate quickly parameters such as center of mass, geometric center, the moment of inertia. Precisely these concepts help to formalize such notions like dynamics and symmetry. To determine the static or dynamic nature of composition, we introduce a quantitative measure of dynamism based on the symmetry of the motif and pattern repeat (rapport) construction.

Static/dynamic nature. When considering the concept of static nature, it seems to be practical to separate the problems of determining the static character of motif and composition in general. However, it is known that certain compositions with dynamic motifs can give a static image. Nevertheless, the dynamic characteristics of motif play an important role in determining the dynamic nature of composition.

According to [1] one of the main characteristics of static nature of motif is the presence of

vertical and horizontal planes of symmetry. To verify this fact, we introduce some concepts that are known from the theoretical mechanics [3].

1. Mass point.There are several ways to determine the mass. The first method - to set the mass equal to $m = 1$ for all points of the motif, the color of which differs from the background color, taking into account the threshold of tolerance.

The second method, which takes into account color, is as follows. The first step is to set the saturation in all the colors of motif to be equal to zero (the background colors are to be excluded from consideration). Thereby, the drawing becomes painted in shades of gray. Then, the mass point is taken proportional to its gray intensity $m = b$ for dark tones of background or $m = 1 – b$ for light tones, provided that the brightness is measured in the range from 0 to 1.

The third way - to deduct the background color from all colors. Then, the design is weighted by the second method as if the background was black.

2. Center of mass.Once mass of points is identified, we can determine the mass of the motif and its center of mass - the point in relation to which the masses are uniformly distributed. Coordinates X, Y of this point are defined as follows:

$$X = \frac{\sum_{i=1}^{N} m_i x_i}{M},\ Y = \frac{\sum_{i=1}^{N} m_i y_i}{M}, M = \sum_{i=1}^{N} m_i ,$$

M– mass motive, N– number of motif points.

3. Moment of inertia. Axial moments of inertia are defined by the following relations:

$$J_{xx} = \sum_{i=1}^{N} m_i (y_i - Y)^2 ,\ J_{yy} = \sum_{i=1}^{N} m_i (x_i - X)^2 .$$

These values show how the image is distributed in relation to the horizontal axispassing through the center of mass J_{xx} and in relation to the vertical axis J_{yy} The larger is the moment of inertia, the farther from the axis of points of image move away. Therefore, if $J_{xx} > J_{yy}$, we can conclude that the figure, in general, along the y-axis has a greater extent than along the x-axis.

The following relation defines the centrifugal moment of inertia:

$$J_{xy} = J_{yx} = \sum_{i=1}^{N} m_i (x_i - X)(y_i - Y) .$$

Now we can consider the matrix, which is also called in mechanics "tensor of inertia"

$$J = \begin{pmatrix} J_{xx} & -J_{yx} \\ -J_{xy} & J_{yy} \end{pmatrix}.$$

From analytic geometry [2] it is known that this matrix is a quadratic form and sets the second-order curve - an ellipse, called in our case the ellipse of inertia. It is also known that we can choose an orthogonal transformation, resulting in the diagonal form of inertia tensor. In this case, the axis of the new coordinate system coincide with the principal axes of the ellipse of inertia.

$$J_0 = \begin{pmatrix} J_x & 0 \\ 0 & J_y \end{pmatrix},\ J_0 = Q^T J Q .$$

The matrix Q is orthonormal matrix defining the rotation of principal axes relative to the x and y axes:

$$Q = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}.$$

To find the principal axes, it is necessary to solve the characteristic equation:

$$\det \| J - \lambda E \| = 0 .$$

E – matrix with ones on the diagonal and zeros beyond it. This is a quadratic equation, solution of which λ_1, λ_2 – eigenvalues of matrix J. Eigenvector (x_1, y_1) – this a single vector components of which are the solution to a system of equations

$$\begin{pmatrix} J_{xx} - \lambda_1 & -J_{yx} \\ -J_{xy} & J_{yy} - \lambda_1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 0 .$$

The second eigenvector is orthogonal to the first and solves a similar equation with a second eigenvalue. We assume that the principal axes are chosen that way that $J_x \geq J_y$. Then we can consider the following parameters.

1. Relation $1 - J_Y/J_X$. This number is equal to zero with the same mass distribution relative to the principal axes; and it is equal to one when the entire design is on one stright line.

2. Sine of double rotation angle of the principal axes $\sin 2\phi$. This parameter is equal to 0 for the coincidence of principal axes with vertical and horizontal directions and equal to one in the diagonal direction.

Thus, we can introduce a parameter of the equilibrium dynamics of motif:

$$Dyn_e = \left(1 - \frac{J_Y}{J_X}\right)\sin 2\varphi \cdot 100\% .$$

It is known that if the body has a symmetry axis, then this axis must be one of the principal axes of the ellipse of inertia. Therefore, for the purpose of diagnostics of -symmetry, we need to check whether the principle axis are symmetry axes. For this purpose, each point in the system of coordinates of principal axes with coordinates(X,Y) must correspond to the same point with coordinates (-X,Y) and/or (X,-Y).

Since the symmetry can be not strict, you should set the threshold of tolerance - the search radius of pair. This number should be small and chosen by the designer. The found pairs are to be memorized. One and the same point must not serve as a point for several others. Now we can introduce symmetry measure along this axis, as a relation of the number of points without a pair to the total number of points of the motif image:

$$Sym = \frac{N_{пр}}{N} 100\% .$$

The introduced measures of equilibrium dynamics and symmetry are notuniversal parameters describing the dynamics of a motif, but they can greatly help the designer in sorting motifs. As mentioned above, two dynamic motifs may, however, create a static composition, balancing each other. On the other hand, static motifs in choosing a nonuniform grid of design repeat (rapport) can give a dynamic picture. We propose a method for estimation of total dynamics of several motifs. For this purpose all the parameters are to be defined which are needed to compute the equilibrium dynamics and symmetry, within the same design repeat (rapport) like for a single motif. Precisely they will give characteristic of dynamics to composition as a whole. It should be noted that these arguments are suitable for any number of motifs.

C O N C L U S I O N S

Quantitative characteristics and assessments to determine measures of dynamics of textile ornament composition have been obtained in this article.

B I B L I O G R A P H Y

1. *Kozlov V.N.* Fundamentals of Textile Products Decoration: Study book for universities. - M., 1981.

2. *Alexandrov P.S.* Course of Analytical Geometry and Linear Algebra.– M., Nauka, 1979.

3. *Tatarinov Y.V.* Lectures on Classical Dynamics. - M.: Publishing office of Moscow university, 1984.

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