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SOLUTION OF THE PROBLEM REGARDING POSITIONS OF PARALLEL STRUCTURE SPHERICAL MANIPULATOR

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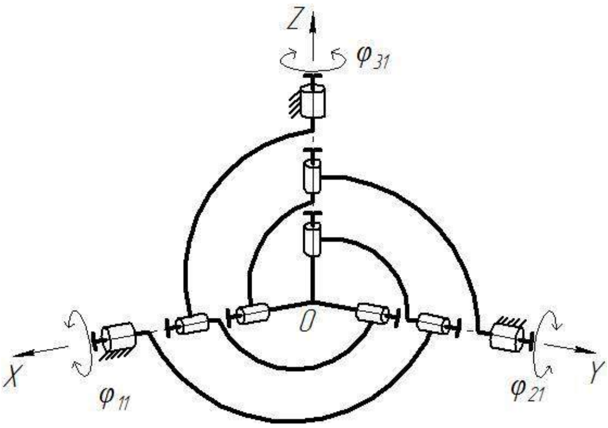
This paper presents kinematics problem of the spherical manipulator parallel structure with three degrees of freedom. Manipulator is consist of three kinematic chain connected to driving mechanisms. Constrain equation is obtained for input and output links. Result of calculations of configurations of the mechanism are defined.

Keywords: spherical parallel mechanisms, determination of configurations, kinematic problem.

This paper is dedicated to research of the parallel three-degree-of-freedom spherical mechanism designed for operation in flexible production systems in textile and light industry [1]. The spherical manipulator under review is illustrated in (Picture 1).



Picture 1. Kinematic diagram of spheritic manipulator

This manipulator comprises three kinematic chains with crossing axes at 90 degree angle. Each input chain link is connected with a rotary actuator. The output link represents a platform rotating around three axes at the point O [2]. Output coordinates are angles of platform rotation: angle α – rotation about the axis x, angle β- rotation about the axis y, angle γ- rotation about the axis z. Generalized coordinates are angles $\phi 11; \phi 21; \phi 31$ - accordingly rotation angles of input links of the first, second and third kinematic chain.

To determine the speed of specific manipulator's positions, problem on positions is to be solved.

Let us write a matrix describing the transition from movable frame to fixed frame of reference of output link. Output link rotates according to the following sequence: rotation about axes: z, y, x.

Rotation matrix about the axis x

$$A1=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\alpha) & -\sin(\alpha)\\ 0 & \sin(\alpha) & \cos(\alpha)\end{pmatrix}.$$

Rotation matrix about the axis y

$$A2=\begin{pmatrix}\cos(\beta) & 0 & \sin(\beta)\\ 0 & 1 & 0\\ -\sin(\beta) & 0 & \cos(\beta)\end{pmatrix}.$$

Rotation matrix about the axis z

$$A3=\begin{pmatrix}\cos(\gamma) & -\sin(\gamma) & 0\\ \sin(\gamma) & \cos(\gamma) & 0\\ 0 & 0 & 1\end{pmatrix}.$$

The matrix describing the transition from movable frame to fixed frame of reference of output link will be of the following form: A=A1*A2*A3.

$$A=\begin{pmatrix}\cos(\gamma)*\cos(\beta) & \cos(\gamma)*\sin(\beta)*\sin(\alpha)-\cos(\alpha)*\sin(\gamma) & \sin(\alpha)*\sin(\gamma)+\cos(\alpha)*\cos(\gamma)*\sin(\beta)\\ \cos(\beta)*\sin(\gamma) & \cos(\alpha)*\cos(\gamma)+\sin(\alpha)*\sin(\beta)*\sin(\gamma) & \cos(\alpha)*\sin(\gamma)*\sin(\beta)-\cos(\gamma)*\sin(\alpha)\\ -\sin(\beta) & \cos(\beta)*\sin(\alpha) & \cos(\beta)*\cos(\alpha)\end{pmatrix}.$$

Input link makes rotation in the first kinematic chain in the following sequence: around the axes x ,y, z.

Rotation matrix about the axis x

$$B1=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\phi11) & -\sin(\phi11)\\ 0 & \sin(\phi11) & \cos(\phi11)\end{pmatrix}.$$

Rotation matrix about the axis y

$$B2=\begin{pmatrix}\cos(\phi11) & 0 & \sin(\phi11)\\ 0 & 1 & 0\\ -\sin(\phi11) & 0 & \cos(\phi11)\end{pmatrix}.$$

Rotation matrix about the axis z

$$B3=\begin{pmatrix}\cos(\phi11) & -\sin(\phi11) & 0\\ \sin(\phi11) & \cos(\phi11) & 0\\ 0 & 0 & 1\end{pmatrix}.$$

Transition matrix from movable frame to a fixed frame of reference of output link will have the following form $B^{/}=B1*B2*B3$.

$$B^{/}=\begin{pmatrix}\cos(\phi12)*\cos(\phi13) & -\cos(\phi12)*\sin(\phi13) & \sin(\phi12)\\ \cos(\phi11)*\sin(\phi13)-\cos(\phi13)*\sin(\phi11)*\sin(\phi12) & \cos(\phi11)*\cos(\phi13)-\sin(\phi11)*\sin(\phi12)*\sin(\phi13) & -\cos(\phi12)*\sin(\phi11)\\ \sin(\phi11)*\sin(\phi13)-\cos(\phi11)*\cos(\phi13)*\sin(\phi12) & \cos(\phi13)*\sin(\phi11)+\cos(\phi11)*\sin(\phi12)*\sin(\phi13) & \cos(\phi11)*\cos(\phi12)\end{pmatrix}$$

Vector coordinates of output link of the first chain have the following coordinates $\begin{pmatrix}0\\0\\1\end{pmatrix}$.

Let us set up equation constraints - $A*\begin{pmatrix}0\\0\\1\end{pmatrix}=B^{/}*\begin{pmatrix}0\\0\\1\end{pmatrix}$ from which we will find angles $\phi11, \phi12, \phi13$ expressing them as α, β, γ.

$$\begin{pmatrix} \sin(\gamma)\sin(\alpha)+\cos(\alpha)\cos(\gamma)\sin(\beta) \\ \cos(\alpha)\sin(\gamma)\sin(\beta)-\cos(\gamma)\sin(\alpha) \\ \cos(\beta)\cos(\alpha) \end{pmatrix} = \begin{pmatrix} \sin(\phi12) \\ -\cos(\phi12)\sin(\phi11) \\ \cos(\phi11)\cos(\phi12) \end{pmatrix}$$

Let us now consider the second kinematic chain; the output link has the following coordinates $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. Input link makes rotation in the second kinematic chain in the following sequence: around the axes y, z, x.

Rotation matrix about the axis y

$$B2 = \begin{pmatrix} \cos(\phi21) & 0 & \sin(\phi21) \\ 0 & 1 & 0 \\ -\sin(\phi21) & 0 & \cos(\phi21) \end{pmatrix}.$$

Rotation matrix about the axis z

$$B3 = \begin{pmatrix} \cos(\phi21) & -\sin(\phi21) & 0 \\ \sin(\phi21) & \cos(\phi21) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Rotation matrix about the axis x

$$B1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi21) & -\sin(\phi21) \\ 0 & \sin(\phi21) & \cos(\phi21) \end{pmatrix}.$$

Transition matrix from movable frame to a fixed frame of reference of output link will have the following form $B^{//} = B2\,B3\,B1$.

$$B^{//} = \begin{pmatrix} \cos(\phi12)*\cos(\phi22) & \sin(\phi21)*\sin(\phi23)-\cos(\phi21)*\cos(\phi23)*\sin(\phi22) & \cos(\phi23)*\sin(\phi21)+\cos(\phi21)*\sin(\phi22)*\sin(\phi23) \\ \sin(\phi22) & \cos(\phi22)*\cos(\phi23) & -\cos(\phi22)*\sin(\phi23) \\ -\cos(\phi22)*\sin(\phi12) & \cos(\phi21)*\sin(\phi23)+\cos(\phi23)*\sin(\phi21)*\sin(\phi22) & \cos(\phi21)*\cos(\phi23)-\sin(\phi21)*\sin(\phi22)*\sin(\phi23) \end{pmatrix}$$

Constraint equation has the following form - $A*\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = B^{//}*\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ from which we will find angles $\phi21, \phi22, \phi23$ expressing them as α, β, γ.

$$\begin{pmatrix} \cos(\alpha)\cos(\beta) \\ \cos(\beta)\sin(\alpha) \\ -\sin(\beta) \end{pmatrix} = \begin{pmatrix} \cos(\phi21)\cos(\phi22) \\ \sin(\phi22) \\ -\cos(\phi22)\sin(\phi21) \end{pmatrix}.$$

The output link in the third kinematic chain has the following coordinates $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. The input link executes rotation in the following sequence: around the axes z, x,y.

Rotation matrix about the axis z

$$B3 = \begin{pmatrix} \cos(\phi31) & -\sin(\phi31) & 0 \\ \sin(\phi31) & \cos(\phi31) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Rotation matrix about the axis x

$$B1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi31) & -\sin(\phi31) \\ 0 & \sin(\phi31) & \cos(\phi31) \end{pmatrix}.$$

Rotation matrix about the axis y

$$B2 = \begin{pmatrix} \cos(\phi31) & 0 & \sin(\phi31) \\ 0 & 1 & 0 \\ -\sin(\phi31) & 0 & \cos(\phi31) \end{pmatrix}.$$

The matrix describing the transition from movable frame to a fixed frame of reference of input link will be of the following form $B^{///} = B3\,B1\,B2$.

$$B^{///} = \begin{pmatrix} \cos(\phi31)\cos(\phi32) - \sin(\phi31)\sin(\phi32)^2 & -\cos(\phi32)\sin(\phi31) & \cos(\phi31)\sin(\phi32) + \cos(\phi32)\sin(\phi31)\sin(\phi23) \\ \cos(\phi31)\sin(\phi32)^2 + \cos(\phi32)\sin(\phi31) & \cos(\phi31)\cos(\phi23) & \sin(\phi31)\sin(\phi32) - \cos(\phi31)\cos(\phi32)\sin(\phi32) \\ -\cos(\phi32)\sin(\phi32) & \sin(\phi23) & \cos(\phi32)^2 \end{pmatrix}.$$

Constraint equation has the following form - $A * \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = B^{///} * \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ from which we will find angles $\phi31, \phi32, \phi33$ expressing them as α, β, γ.

$$\begin{pmatrix} \cos(\alpha)\sin(\beta)\sin(\gamma) - \cos(\gamma)\sin(\alpha) \\ \cos(\alpha)\cos(\gamma) + \sin(\alpha)\sin(\beta)\sin(\gamma) \\ \cos(\beta)\sin(\gamma) \end{pmatrix} = \begin{pmatrix} -\cos(\phi32)\sin(\phi31) \\ \cos(\phi31)\cos(\phi32) \\ \sin(\phi32) \end{pmatrix}.$$

The function of mechanism position shows dependence of the output link coordinates from the generalized coordinate. Implicitly, function of mechanism's position is expressed by the expression

$$F(\alpha, \beta, \gamma, \phi11, \phi12, \phi13) = 0.$$

Constraint equation for spherical manipulator can be expressed by the following system of equations:

$$\begin{cases} F_1(\alpha, \beta, \gamma, \phi11) = 0 \\ F_2(\alpha, \beta, \gamma, \phi12) = 0 \\ F_3(\alpha, \beta, \gamma, \phi13) = 0 \end{cases}$$

We will put the angles expressed $\phi11, \phi21, \phi31$ as α, β, γ into the equations of constraints:

$$\begin{cases} F_1 = tg(\phi11) - \dfrac{\cos(\gamma)\sin(\gamma)\sin(\beta) + \cos(\gamma)\sin(\alpha)}{\cos(\alpha)\cos(\beta)} = 0 \\ F_2 = \dfrac{\sin(\beta)}{\cos(\gamma)\cos(\beta)} - tg(\phi12) = 0 \\ F_3 = \dfrac{\cos(\gamma)\sin(\beta)\sin(\alpha) - \cos(\alpha)\sin(\gamma)}{\cos(\alpha)\cos(\gamma) + \sin(\alpha)\sin(\beta)\sin(\gamma)} + tg(\phi13) = 0 \end{cases}$$

Thus, equations of constraints are set up between rotation angles of input links and rotation angles of output links.

Let us consider an example of inverse solution on positions. We need to determine the generalized coordinates (rotation angles of input links $\phi11, \phi21, \phi31$), with known $\alpha = 1рад, \beta = 1рад, \gamma = 1рад$. In this case rotation angles of input links will be: $\phi11 = 0,242рад$, $\phi21 = 1,237рад$, $\phi31 = 0,081рад$.

CONCLUSION

1. Equations of constraints between the input and output links of manipulator have been set up.
2. Problem on positions of spherical mechanism has been solved

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