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THEORY AND CURVE PLOTTING OF BIAXIAL STRETCHING OF FABRIC

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Theory of biaxial stretching of fabric with consideration for extensibility of warp and weft threads, as well as change of fabric structure in deformation has been presented. Problem of deformation analysis and loads appearing with extension of fabric with a pre-determined fabric structure, set of warp and weft, linear density of threads, their deformation characteristic has been set out and solved.

The fabric biaxial stretching theory in a view of basis and duck strings extensibility and fabric deformation structure changes is represented. The problem of calculation of deformations and stretching fabric loadings is put and solved at the given fabric structure, density on a basis and a duck, linear density of strings and their deformation characteristics.

Keywords: thread, fabric, warp, weft, extension, tension, nonlinear mechanics, rigidity.

Theory of biaxial stretching of fabric with consideration for extensibility of warp and weft threads is presented in this paper. Problem of deformation analysis and loads appearing with extension of fabric with a pre-determined fabric structure, set of warp and weft, linear density of threads, their deformation characteristic has been set out and solved. Thus, it is assumed that lateral dimensions do not change with deformation of fabric. Of course, cross-section of thread even in the unloaded fabric is not round in the interaction zone of beam and filling thread that can be easily seen on the transverse section of fabric. Change of diameter of threads occurs due to tension of threads and compressive forces in the crossing zone of warp and weft threads. Threads are subjected to significant loads during formation of fabric on the weaving loom, especially in the course of battening, when the reed presses into the weft between the strong tensioned threads of warp. Formation of fabric is designed in such a way, so that the produced structure should remain even after removal of fabric from the machine. That's why the term 'strained section of threads' is accepted in calculations, dimensions of which are determined from cuts of fabric under investigation. There is intention to elaborate in future the discussed herein theory further, with consideration for compressibility of threads.

Fabric will be considered in two conditions.

1) When fabric is in free conditions, i.e there is no external load. In this case, one

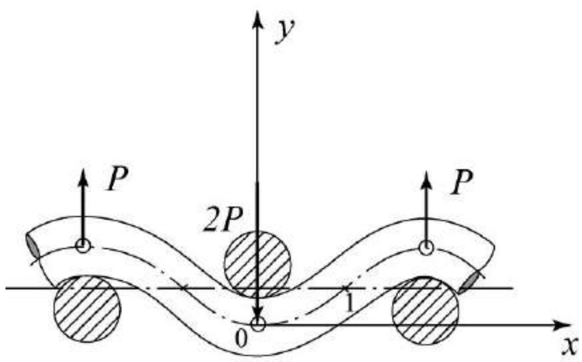
should bear in mind that a real fabric exists as a single-piece physical flat object (body) due to interaction of warp and weft threads. Attempt of an elastic thread, curved during weaving, to restore its natural straight form results in occurrence of forces acting in the zone of crossing threads. Resultant force of the distributed here ones - is a force that is a result of interaction of two threads coming in touch one with another. Parameters, which determine the stress-strain state of an unloaded fabric, depend significantly from the rigidity of warp and weft threads. Methods of strength of materials are not applicable due to big curvature of both thread systems; therefore description of thread's condition will be made with the aid of methods of the geometrically nonlinear theory of elastic thread, where axial line of thread is taken as inextensible.

2) The fabric under the tensile loads directed along the warp and weft threads. Tension of thread occurring with deformation of thread straightens the curved thread. Real structure is replaced with a model in which axes of warp and weft – straight lines with the bending point in the center of overlapping of threads. Strain in the thread σ_f caused by its stretching is related to deformation ε_f by exponential law

$$\sigma_f = E\varepsilon_f^m. \qquad (1)$$

Considering strain as an internal force, imposed on fiber and not on threads with free space between fibers, we should notice that it is incorrect to determine the strain in the thread and yarn as relation of force to the sectional area of yarn (thread) $\pi d^2/4$ since this load is absorbed not only by fibers but also by air pockets in the yarn; they are embraced in this formula too. Only area of fibers is to be considered which fall within the cross-section of the yarn taking into account both location of fibers at an angle with yarn axis, and different fiber orientation in radial direction. It is preferable to use specific stress as a force related to the mass of unit length. Specific stress unit is 1 N/tex. Relation between the normal stress and the σ specific one σ_s, density of yarn ρ is expressed by the formula $\sigma = \sigma_s\rho$. Taking as a density unit ρ g/sm^3, we get stress unit which equals to 1 GPa: stress, 1 GPa = specific stress, 1 N/Tex × yarn density, g/sm^3.

Let us consider the first state of yarn, when there is no load. Design diagram is given in Picture 1.



Picture 1

The length of elastic curve is unknown, but the distance between the support is defined. Rigidity of yarn is defined by the methods [1]. The distributed load acting on the length ℓ and which represents a weight of yarns bended under the applied force at the free end of console P_1, substitute of resultant force P_2. With measured length, experimentally defined weight of yarn combined with concentrated weight of a plate and measured coordinate of plate's center of gravity from the system given below, rigidity of all bended threads H_Σ, modules of elliptical integrals k_1, k_2 and their amplitude are computed α_{01}, α_{11}, α_{02}:

$$\begin{gathered}\alpha_{12} = 90^\circ,\ k_1 \sin\alpha_{01} = 0{,}707,\\ F(\alpha_{11}) - F(\alpha_{01}) = \omega_1,\ \langle F(k_2) - F(\alpha_{02}) = \omega_2 \rangle,\\ k_2\cos\alpha_{02} = k_1\cos\alpha_{11},\ k_2\sin\alpha_{02} = k_1\sin\alpha_{11}.\end{gathered} \qquad (2)$$

In this case, tracing of extension curve is carried out on the example of a technical fabric, article 86494-05, produced out of high-modulus yarn "Rusar" with linear density 29 tex. Bending rigidity of yarn equals to H = $=9{,}82 cN \cdot mm^2$. Our task is focused on considering bending of console elastic yarn at the zone 01 (Picture 1) [1], [2]. With yarn density on set of warp P_o = 260 yarn/dm and the same set of weft coordinates x_1 = 0,192 mm and y_1 = 0,08 mm. The second coordinate is defined by sections of fabric and in size differs significantly from diameter of initial yarn, that was mentioned in the first paragraph. The length of axial line of yarn ℓ is unknown and is to be defined, as well as angle φ between the axle of yarn and the neutral line of structure in point 1. Equations for calculation in four unknown ℓ, P, k, α_0 are given in [1], [2]:

$$\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-k^2\sin^2\alpha}} d\alpha - \int_0^{\alpha_0} \frac{1}{\sqrt{1-k^2\sin^2\alpha}} d\alpha = \sqrt{\frac{P\ell_0^2}{H}}, \quad k\sin\alpha_0 = \frac{\sqrt{2}}{2},$$

$$\frac{y_1}{\ell_0} = 1 - \frac{2}{\sqrt{\frac{P\ell_0^2}{H}}} \left[\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-k^2\sin^2\alpha}} d\alpha - \int_0^{\alpha_0} \frac{1}{\sqrt{1-k^2\sin^2\alpha}} d\alpha \right], \qquad (3)$$

$$\frac{x_1}{\ell_0} = \frac{2}{\sqrt{\frac{P\ell_0^2}{H}}} k\cos\alpha_0 .$$

The length of axial line of zone 01 equals to 0.211 mm. In this case we can compute the run-in of beam thread

$$a_o = \frac{\ell_0 - x_1}{\ell_0} \times 100 = \frac{0{,}211-0{,}192}{0{,}211} \times 100 = 9{,}0\,\%. \quad (4)$$

The inclination φ angle of tangential at the end point 1

$$\varphi = 2\arcsin k - \frac{\pi}{2} = 0{,}585\,\text{rad} = 33{,}5^\circ. \quad (5)$$

Special attention is to be paid to force value of interaction of threads at the contact zone: 2P=2·294,68 mN=2·29,468 sN≈58,9 gs. Such significant pressing force of threads on each other, which is equal to 58,9 cN, form a stabile structure of fabric, its ability to withstand external impacts. It is determined first of all by significant yarn's bending rigidity H = $9{,}82\ cN \cdot mm^2$. For comparison we will state the value N of cotton yarn of the same linear density – $0.23\ cN \cdot mm^2$ [3] (43 times less). Besides, for movement of a free yarn end to the point with the coordinate y_1=0,08 mm with high degree of rigidity a significant force is needed. Let us calculate, what error we commit, when we accept geometrically linear bending theory with the same yarn characteristic. In this case, the value

$$y_{1L} = \frac{Px_1^3}{3H} = \frac{29{,}468 \times 0{,}192^3}{3 \times 9{,}82} = 7{,}08 \times 10^{-3}. \quad (6)$$

Relative error $\delta = \frac{0{,}08-0{,}00708}{0{,}08} \times 100 = 91{,}2\,\%.$

The figgure comments itself.

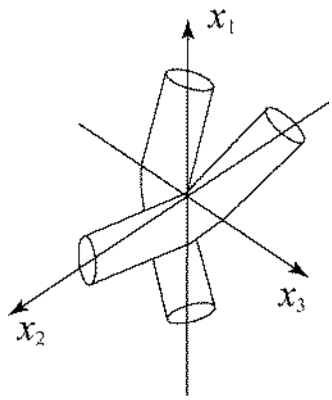
Let us look now at biaxial extension of fabric. Theory of elastic threads is not applicable to this problem since inextensibility of yarn's axle is supposed to be one of main hypotheses of this theory. As mentioned above, thread tension, occurring with yarn stress, straightens the bended thread; then warp and weft threads represent straight lines with the bending point at the center of overlap of threads. Full image about thread properties with uniaxial tension gives us the tension diagram σ≈ε (the term "diagram" is accepted widely in the whole technical literature). Tests of "Rusar" thread have been carried out on the machine FP-100/1. Test data of stress σ, GPa and deformation ε are presented in a vector form:

$$\sigma = [0{,}726\ \ 1{,}403\ \ 1{,}964]^T,$$
$$\sigma = [0{,}01\ \ 0{,}02\ \ 0{,}029]^T.$$

This record is accepted for approximation of elongation curve by power function $\sigma=E\varepsilon^m$, by methods of numerical optimization. The generally accepted method in solving problems or smoothing method is the following one: $\sum_{i=1}^{n}\left(\sigma_{iэ}-E\varepsilon_i^m\right)^2$ ®min. Solution could be E = 52,756 GPa and m=0,929.

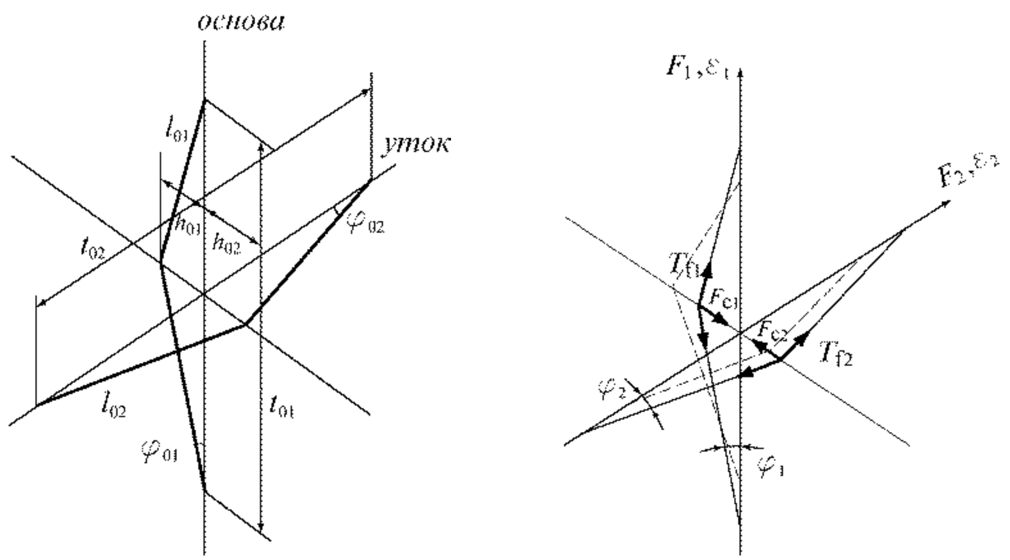
If we approximate experimental data with the aid of linearization method, as it is usually done, when nonlinear function is linearized, and then selection of parameters of linear function is carried out, then the difference of values of power function parameters which are calculated by two methods, is significant. The values obtained by optimization method are more trustworthy since in the case of linearization the selected estimation does not meet requirements of efficiency.

The outlined below biaxial fabric's extension theory is based on the paper of Kawabata, Niwa, Kawai [4];of course, it will not give a complete review of thereof. When prof. Kawabata only describes the mechanics of plain weave fabric, setting warp and weft values, we have calculated it with the aid of theory of real elastic thread (formula (4)) by determining the length of the axial line of a section (zone) 01. Later we will also outline other specific of our theory.



Picture 2

Let us consider biaxial thread extension along the axes x_1 and x_2 (Picture 2). Let us denote the length (step) of structural element along x_1 through t_{01}, along x_2 – through t_{02} (Picture 3-a).



a) b)

Picture 3

Later, index 0 will correspond to the undeformed state, index 1 – to the value related to wart threads, index 2 – to weft threads. The step is related to thread density P (number of threads per 10 decimeter) with ratio

$$t_{0i}=\frac{100}{P_i}\ (i=1,2). \qquad (7)$$

At thread boundaries extensile force operates f_i. If for the fabric length that much n_i threads are accounted for, then at the end of one thread along the axis x_i the force $F_i = f_i/n_i$ is involved (Picture 3-b). Under tension of fabric, along the both axes occurs tension T_{fi} related to the force exerted on fabric by relation

$$F_i=T_{fi}\cos\varphi_i, \qquad (8)$$

where φ_i – angle between the axis of thread and neutral line of structure in the deformed state (Picture 3-b). In the simulating of fabric tension we suppose that the distance between warp thread and weft thread at their contact line remain constant on the assumption when transverse size of threads remains unchanged:

$$h_1+h_2=h_{01}+h_{02}. \qquad (9)$$

Let us be reminiscent of the fact that in the paper problem of calculation of deformations and forces appearing with a pre-determined fabric structure, set of warp and weft P_i, linear density of threads T_i, their deformation characteristic has been set out and solved. Thread tension is determined by its deformation according to expression $\sigma = E\varepsilon^m$

$$T_{fi}=E_i\varepsilon_{fi}^m\frac{T_i}{\rho_i}. \qquad (10)$$

Fabric deformation ε_i is related to thread deformation ε_{fi} by dependence

$$\varepsilon_{fi}=\frac{\ell_i}{\ell_{0i}}-1=\frac{\frac{t_{0i}}{2}\sqrt{\left(\frac{2h_i}{t_{0i}}\right)^2+\left(1+\varepsilon_i\right)^2}}{\ell_{0i}}-1. \qquad (11)$$

It was noted above that real structure is replaced with a model in which axes of warp and weft – straight lines with the bending point in the center of overlapping of threads. Form and length of thread in the structural element are already determined in the first part of paper where free state of fabric was considered. This thread length ℓ_{0i} = 0,211 mm is tensioned under loading of fabric. Bended thread is straightened. With initial thread length ℓ_{0i} and the step of structural element t_{0i}, distance between the neutral line of structure and the thread axis along the axis x_3 at the bending point is calculated:

$$h_{0i}=\sqrt{\ell_{0i}^2-\left(\frac{t_{0i}}{2}\right)^2}=0{,}085\ \text{mm}.$$

Weft thread is in equilibrium under action of tensile forces T_{f2} and reaction N of back warp thread directed along the axis x_3 at the contact point of warp and weft threads. On back warp thread reaction of N' weft thread is exerted which is equal on module and is oriented N in the opposite direction of force. Both reactions are balanced by press force of threads on each other which press threads and cause displacement of both threads systems along the axis x_3 (Picture 3-b). In deformed state $h_2=h_{01}+h_{02}-h_1$. Pressing force is related to thread tensioning by relationship $F_{ci}=2T_{fi}\sin\varphi_i$. Here

$$\sin\varphi_i = \frac{h_i}{\sqrt{h_i^2 + \left[\frac{(1+\varepsilon_i)t_{0i}}{2}\right]^2}} = \frac{2h_i/t_{0i}}{\sqrt{\left(\frac{2h_i}{t_{0i}}\right)^2 + (1+\varepsilon_i)^2}}. \quad (12)$$

From equality $F_{c1} = F_{c2}$ and relations (11) we get an expression for determination $h_1, \varepsilon_{f1}, \varepsilon_{f2}$:

$$\left(E_1\varepsilon_{f1}^{m_1}\frac{T}{\rho}\right)\left[\frac{4h_1/t_{01}}{\sqrt{\left(\frac{2h_1}{t_{01}}\right)^2 + (1+\varepsilon_1)^2}}\right] = \left(E_2\varepsilon_{f2}^{m_2}\frac{T}{\rho}\right)\frac{4(h_{01}+h_{02}-h_1)/t_{02}}{\sqrt{\left(\frac{h_{01}+h_{02}-h_1}{t_{02}}\right)^2 + (1+\varepsilon_1)^2}}. \quad (13)$$

By the formula (10) we calculate tension of warp and weft thread. With consideration for $\varphi_i = \mathrm{arctg}\left[\frac{2h_i}{(1+\varepsilon_i)t_{0i}}\right]$ we determine the force stretching the fabric F_i:

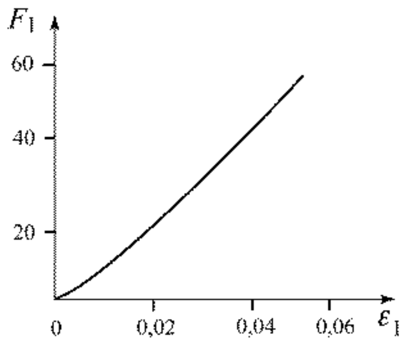
$$F_i = T_{fi}\cos\varphi_i. \quad (14)$$

Let us, as an example, consider tension of thread in direction of warp with fixed deformation in direction of weft. In this case ε_2 remains equal zero under tension of fabric.

Below, computed values F_1 of the force stretching the fabric are given, related to one back warp thread as well as to deformation of fabric in direction of warp ε_1:

$$\varepsilon_1 = [0\ 0{,}01\ 0{,}02\ 0{,}025\ 0{,}03\ 0{,}04\ 0{,}05\ 0{,}055]^T,$$
$$F_1 = [0\ 10{,}05\ 19{,}94\ 25{,}01\ 30{,}19\ 40{,}85\ 51{,}93\ 57{,}61]^T.$$

Fabric deformation value $\varepsilon_1 = 0{,}05$ corresponds to deformation of warp thread $\varepsilon_{f1}=0{,}025$ which is located close to finite strain 0.029. Fabric extension diagram $F_1:\varepsilon_1$ in direction of warp with $\varepsilon_2 = 0$ is illustrated in Picture 4.



Picture 4

Thus, description of tension of plain fabric has been received. The theory outlined herein could be used in the theory of other structures, such as serge, sateen, etc. Experimental tests is expected to be carried out on a device which is being developed at the Kostroma university of technology; as authors of this paper assert, any fabric loading law can be simulated on this device, including the load which simulates formation of fabric on a loom. Finally, another remark. Of course, it is easier from technical point of view to obtain fabric tension diagram on a finished sample. But if we turn to structural mechanics, as it was done herein, we can find relations which comprise main characteristic of thread and parameters determining the structure of cloth. Therefore, the suggested theory enables to explain the mechanism of phenomenon under deformation and loading of cloth, as well as to control the weaving process with tailor-made properties.

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